PRESBYTERIAN LADIES' COLLEGE

A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT
MATHEMATICAL METHODS YEAR 12 – TEST 2

DATE: 2nd March 2016 Name: Solutions

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE
WORKING TIME: Maximum 30 minutes
TOTAL: 31 marks
EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet (provided)

SECTION TWO: CALCULATOR ASSUMED
WORKING TIME: Minimum 20 minutes
TOTAL: 19 marks
EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 calculators, formula sheet (provided), one A4 page of notes (one side only)

Question	Marks available	Marks awarded	Question	Marks available	Marks awarded
1	19		4	7	
2	6		5	6	
3	6		6	6	
Sect 1 Total	**31**		**Sect 2 Total**	**19**	
			TOTAL	50	

{−1 no +c for whole test}

Question 1 **(19 marks)**

(a) Determine the following, writing your answers with positive indices and simplifying where possible.

(i) $\frac{dy}{dx}$ if $y = x^3 + e^{2x}$ (2 marks)

$\frac{dy}{dx} = 3x^2 + 2e^{2x}$ ✓ ✓

(ii) $\int 8xe^{x^2}\,dx$ (2 marks)

$= 4e^{x^2} + c$ ✓ ✓

(iii) $\int \left(e^{2x} + e^{-2x}\right)^2 dx$ (3 marks)

$= \int (e^{2x} + e^{-2x})(e^{2x} + e^{-2x})\,dx$

$= \int (e^{4x} + 2e^{0} + e^{-4x})\,dx$ ✓

$= \frac{e^{4x}}{4} + 2x + \frac{e^{-4x}}{-4} + c$ ✓

$= \frac{e^{4x}}{4} + 2x - \frac{1}{4e^{4x}} + c$ ✓

Question 1 continued on next page …

Question 1 continued …

(iv) $\dfrac{dy}{dx}$ if $y = xe^{2x}$ (2 marks)

$\dfrac{dy}{dx} = x2e^{2x} + e^{2x}$ ✓ ✓

$= e^{2x}(2x+1)$

No need to factorise for final answer.

(b) (i) Determine $\dfrac{d}{dx}\int_1^x 8t(t^2-2)^3\,dt$ (2 marks)

$= 8x(x^2-2)^3$ ✓✓

(ii) Evaluate exactly $\int_0^1 \dfrac{e^{3x}-e^{2x}}{e^x}\,dx$ (4 marks)

$= \int_0^1 e^{2x} - e^x \,dx$ ✓

$= \left[\dfrac{e^{2x}}{2} - e^x\right]_0^1$ ✓

$= \dfrac{e^2}{2} - e - \left(\frac{1}{2} - 1\right)$ ✓

$= \dfrac{e^2}{2} - e + \dfrac{1}{2}$ ✓

f/t substitution if anti-diff incorrect so long as not too easy. (2)

Question 1 continued on next page …

Question 1 continued …

(iii) Evaluate exactly $\int_{\alpha}^{2\alpha}\left(pe^{px+q}+2pxe^{px^2}\right)dx$ (4 marks)

$$= \left[e^{px+q} + e^{px^2}\right]_{\alpha}^{2\alpha}$$

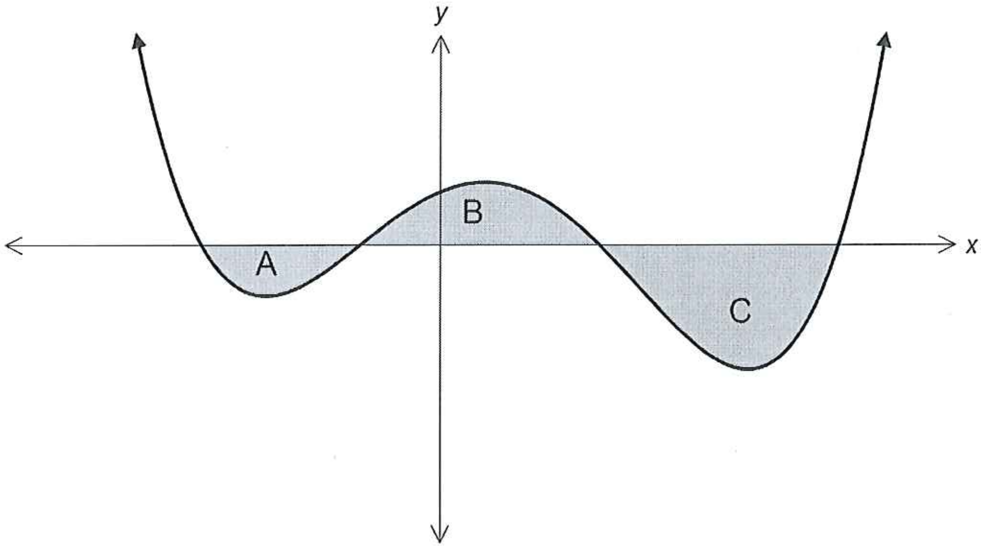
Full marks here okay

$$= e^{p(2\alpha)+q} + e^{p(2\alpha)^2} - \left(e^{p\alpha+q} + e^{p\alpha^2}\right)$$

$$= \underbrace{e^{2p\alpha+q} + e^{4p\alpha^2}}_{\checkmark} \underbrace{- e^{p\alpha+q} - e^{p\alpha^2}}_{\checkmark}$$

Question 2 **(6 marks)**

Consider the following function, $y = f(x)$, with x-intercepts at -3, -1, 2 and 5.



The area of A is 3 cm^2, B is 6 cm^2 and C is 10 cm^2.
Use the graph above to determine the following definite integrals.

(a) $\int_{-3}^{-1} f(x)\,dx$ $= -3$ ✓ (1 mark)

(b) $\int_{-3}^{5} f(x)\,dx$ $= -3 + 6 - 10$ (1 mark)

$= -7$ ✓

(c) $\int_{-3}^{5} -f(x)\,dx$ $= 3 - 6 + 10$ (1 mark)

$= 7$ ✓

(d) $\int_{-1}^{2} (f(x) + 4)\,dx$ $= 3 \times 4$ ✓ $+ 6$ (2 marks)

$= 18$ ✓

(e) $\int_{-5}^{1} f(-x)\,dx$ $= 6 - 10$ (1 mark)

$= -4$ ✓

Question 3 **(6 marks)**

A curve has equation $y = x^2 e^{-x}$. Show that $\frac{dy}{dx} = ax^b e^{-x}(c - x)$, giving the values of a, b and c.

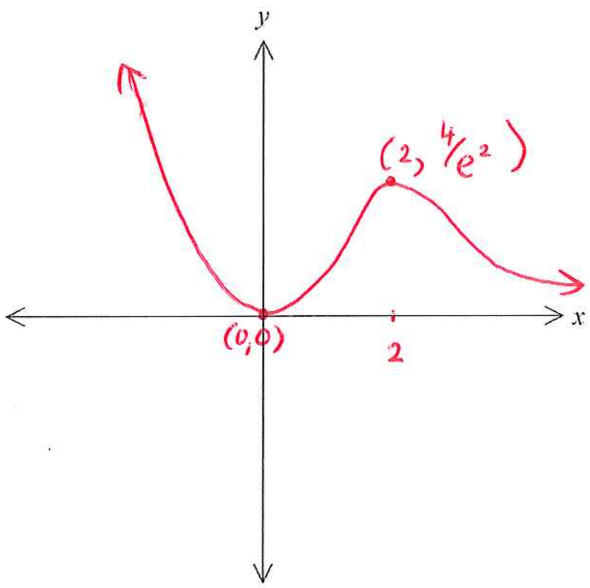
Sketch the graph on the axes below. Clearly show the exact location of any turning points, intercepts and asymptotes.

$$\frac{dy}{dx} = x^2(-e^{-x}) + 2xe^{-x}$$
$$= xe^{-x}(2 - x) \checkmark$$

$a = 1$, $b = 1$, $c = 2$ ✓

St. Pts

$$0 = xe^{-x}(2-x)$$
$$\begin{cases} x = 0 \\ y = 0 \end{cases} \text{ or } \begin{cases} x = 2 \\ y = \frac{4}{e^2} \end{cases} \checkmark$$



As $x \to \infty$, $y \to 0$

✓shape

✓TP's

✓asymptote

End of Section 1

Section 2: Calculator Assumed **Name:** ______________________

Question 4 **(7 marks)**

Students at the University of Sydney observed the number of possums in a nearby area of bushland. It was known that the original population when they commenced observation in January 2010 was approximately 250. The population of possums was found to be growing such that $\frac{dP}{dt} = 0.05P$.

(a) Write an equation that can be used to determine the number of possums, t years after the initial observations by the students. (2 marks)

$P = 250e^{0.05t}$ ✓ ✓

(b) Determine the population of possums in July 2015. (2 marks)

$t = 5.5$ ✓

$P = 250e^{0.05(5.5)}$

$= 329.1326\ldots$

$\therefore$ 329 possums ✓ (330 okay).

-1 if .13 included

(c) Determine, to the nearest month, when the number of possums will first exceed 400. (3 marks)

$400 = 250e^{0.05t}$ ✓

$t = 9.4$ years ✓

$\therefore$ may 2019 ✓

9 yrs 5 months okay

Question 5 **(6 marks)**

The amount of current in a circuit, $I(t)$ amps, decreases in accordance with the rule $\frac{dI}{dt} = \frac{-100}{t^2}$ where t is the time in seconds, provided that $t \geq 0.2$ seconds. It is known that when $t = 2$, the current is 150 amps.

(a) Determine a formula for the current at any time, $t \geq 0.2$ seconds. (2 marks)

$$\frac{dI}{dt} = -100t^{-2}$$

$$I = \frac{-100t^{-1}}{-1} + c$$

$$I = \frac{100}{t} + c$$

$t = 2,\ I = 150$

$\Rightarrow c = 100$

$\therefore I = \frac{100}{t} + 100$

(b) Find the current after 20 seconds (1 mark)

$$I = \frac{100}{20} + 100$$

$= 105$ amps

(c) Determine the amount of current lost during the fifth second. (2 marks)

$$I(4) = \frac{100}{4} + 100$$

$= 125$ $\frac{1}{2}$

$$I(5) = \frac{100}{5} + 100$$

$= 120$ $\frac{1}{2}$

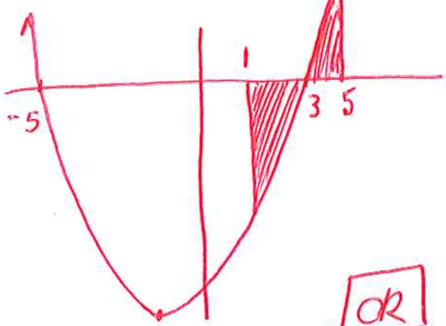
$\therefore$ 5 amps lost in 5^{th} second

(d) Describe what happens to the current as $t \to \infty$. (1 mark)

As $t \to \infty$, $I \to 100$

Question 6 **(6 marks)**

(a) Find the area bound by the curve $y = x^2 + 2x - 15$ between $x = 1$, $x = 5$ and the x-axis. (3 marks)



$$\int_1^5 |x^2 + 2x - 15|\, dx$$

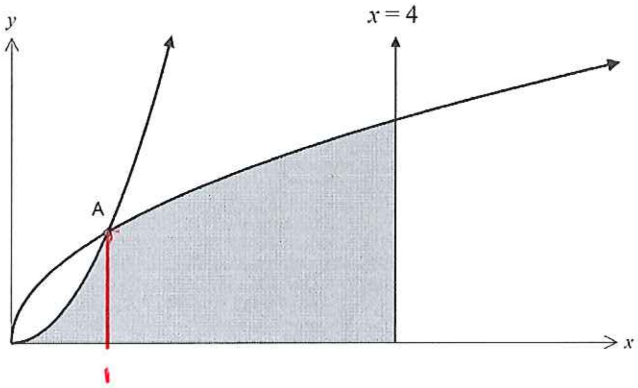
$$= 32 \text{ units}^2$$

$\boxed{\text{OR}}$ $$-\int_1^3 x^2 + 2x - 15\, dx + \int_3^5 x^2 + 2x - 15\, dx$$

$$= 13\tfrac{1}{3} + 18\tfrac{2}{3}$$

$$= 32 \text{ units}^2$$

(b) The curves $y = ax^2$ and $y = a\sqrt{x}$ intersect at the point A (1, a) as shown in the diagram below.



If the shaded area is equal to one square unit, find the value of a. (3 marks)

$$\int_0^1 ax^2\, dx + \int_1^4 a\sqrt{x}\, dx = 1$$

$$\left[\frac{ax^3}{3}\right]_0^1 + \left[\frac{2ax^{3/2}}{3}\right]_1^4 = 1$$

NOTE
Can
Solve on
CAS

$$\frac{a}{3} + \frac{16a}{3} - \frac{2a}{3} = 1$$

$$a = 0.2$$

End of Test